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ORIGINAL CONTRIBUTION

Effect of Awareness and Treatment on COVID-19 Pandemic Mathematical Model

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ABSTRACT

In this paper, I have considered a six-dimensional COVID-19 pandemic mathematical model. I have studied my proposed model analytically and investigated the local stability of both equilibrium points. I have also calculated the basic reproductive number R_0 and its value with the assistance of the next-generation matrix technique. In reality still, now there is no proper treatment procedure. In most countries of the world vaccination process is completed but there is no complete conclusion that if an individual is given a vaccine then there is no chance of being infected by the virus. So in this situation, it is very important to find a way to control the disease transmission. From the analysis of my work, I have established that if I increase the awareness (wearing masks, using sanitizer, avoiding much more gathering, etc.) and treatment rate to a certain value then the disease transmission can be kept under control. I have also justified my analytical results numerically with the help of MATLAB programming.

KEY WORDS: COVID-19 Pandemic model, Stability, Awareness, Sensitivity Analysis, Treatment, Basic reproductive number.

1. INTRODUCTION

In reality, so many mathematicians and scientists are doing work on different kinds of life-killing diseases such as cancer, HIV/AIDS, dengue, Ebola COVID-19, etc. The coronavirus (COVID-19) disease was first identified in Wuhan City the capital of Hubei, China, in December 2019 [1, 2]. But in a very short period spread globally across the World and WHO declared it is a pandemic. Today it is a very important area for mathematicians to do work on biological problems. Mathematical models give truthful strategies for how to prevent such types of life-killing infectious diseases [3-10]. According to Worldometer updates on 26th July, 2023 all over the world total coronavirus confirmed and death cases were 692,146,811 and 6,902,831. All over the world people are facing various troubles from different aspects i.e. economic, educational, health-related, travel, etc.

So many major initiatives were taken by all the health authorities of the world but they did not succeed in preventing the disease transmission. In a very short period, COVID-19 was spread all over the world and changed its nature. Therefore it is very difficult to predict its nature to take initiative steps and provide adequate facilities to the affected people. In India, the first COVID-19 cases were recognized on January 27, 2020, in Kerala.

The etiological agent of COVID-19 is Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2). Actually SARS-CoV-2 name was given by the ICTV (International Committee on Taxonomy of Viruses on February 11, 2020 [11]. All kinds of viruses generally alter their structure and it depends on time. Through mutation SARS-CoV-2 virus has also altered their structure. Now SARS-CoV-2 virus is going through its six strains. According to Xie et.. al. [12] first episode was sprouted in Guangdong Province of China in (2002-2003). In (2012-2020) the second one MERS originated in Saudi Arabia.

The third attack of SARS-CoV-2 has been occurred in 2021 and its fatality rate is lower than MERS-CoV. Every person is treated as susceptible to being affected by the virus. However, older people and those people who are already suffering from some specific

diseases are at high risk of getting COVID-19 disease. Huge mathematicians already worked on and published many papers on COVID-19 [13-21]. In most cases, about 81 percent of COVID-19-affected people are asymptomatic and do not need oxygen support treatment. The basic symptoms of coronavirus-infected individuals are fever, dry cough, tiredness, and breathing problems.

Among the infected individual, 14 percent of acute cases need oxygen support (ICU) treatment when their oxygen saturation level is less than 93 percent (95 to 100 percent is the normal range). COVID-19 is the most effective contagious disease spread human to human through NPIs. To reduce the community spread of coronavirus only lock-down is a reliable effective mechanism [22-24]. Some researchers also used fractional order differential equations to study the COVID-19 disease [25-28].

In this work, I have considered a nonlinear six-dimensional deterministic epidemic model with awareness and treatment and suppose that treatment and recovery classes do not consider the total active population. My main motivation for this work is how awareness and treatment can prevent disease transmission.

2. DIFFERENT FEEDSTOCKS

In this paper, I have considered a six-dimensional COVID-19 pandemic mathematical model. Here I have divided the total population into six different classes such as; $S(t)$, the active population class, $NQ(t)$, the non-quarantine class, $Q(t)$, the quarantine class, $I(t)$, the COVID-infected class, $T(t)$, the treatment class and $R(t)$, release class. In my proposed SNQQITR model, I have assigned a term which is prevent the disease transmission i.e. awareness (wearing masks, using sanitizer, avoiding much more gathering, etc.). Under the above consideration, my desired model is taken in the given form.

Where R_s is the recruitment rate of the active population class, β_1 is the contact rate between S and I class β_2 is the contact rate between S and NQ class, γ is the quarantine period ($\frac{1}{14}$) days, μ is the awareness rate, d is the natural death rate, α is a constant ($0 \leq \alpha \leq 1$), η_1 is the rate in which non-quarantine class goes to COVID

infected class, η_2 is the rate in which quarantine class goes to treatment class, θ is the treatment rate of COVID infected class, ε is release rate due to treatment, δ_1 , and δ_2 is the disease-related death rate.

3. Various Preliminary Discussion of the Model (1)

Theorem 3.0.1 *The solutions $S, N_Q, Q, I, T,$ and R of the model (1) are positive for $\forall t > 0$ in R_+^6 with the assumption of the conditions $S(0) > 0, N_Q(0) \geq 0, Q(0) \geq 0, I(0) \geq 0, T(0) \geq 0$ and $R(0) \geq 0$ in the region Λ .*

Proof: From model (1), we can write $\frac{dS}{dt} /_{S=0} = R_s + \gamma Q, \frac{dN_Q}{dt} /_{N_Q=0} = \alpha\beta_1SI,$
 $\frac{dQ}{dt} /_{Q=0} = (1 - \alpha)\beta_1SI, \frac{dI}{dt} /_{I=0} = \eta_1N_Q, \frac{dT}{dt} /_{T=0} = \eta_2Q + \theta I$ and $\frac{dR}{dt} /_{R=0} = \mu S + \varepsilon T.$

Hence due to the above assumption, all solutions of the SN_QQITR system are positive for all $t > 0$ in Λ .

4. Basic Reproductive number R0

In epidemiology, basic reproductive number plays an important role in spreading disease. I have calculated the basic reproductive number with the help of the next-generation matrix technique [29]. The non-negative matrix, f_i , of the infection terms and the non-singular matrix v_{ni} , of the non-infection terms are

$f_i =$ The infection terms =
$$\begin{pmatrix} \alpha\beta_1SI + \beta_2SN_Q \\ 0 \\ 0 \end{pmatrix}$$

Theorem 3.0.2 *All positive solutions of the system (1) are bounded and belong to the region*

$$\Lambda = \{(S, N_Q, Q, I, T, R) \in R_+^6 : 0 \leq S + N_Q + Q + I + T + R \leq \frac{R_s}{d}\}.$$

Proof: Here I have considered that $N = S + N_Q + Q + I + T + R$. Therefore,

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dN_Q}{dt} + \frac{dQ}{dt} + \frac{dI}{dt} + \frac{dT}{dt} + \frac{dR}{dt} \leq R_s - dN.$$

Hence

$$\limsup_{t \rightarrow \infty} (S + N_Q + Q + I + T + R) \leq \frac{R_s}{d}.$$

So, all solution (positive) of the system (1) is ultimately bounded and belongs to the region

$$\Lambda = \{(S, N_Q, Q, I, T, R) \in R_+^6 : 0 \leq S + N_Q + Q + I + T + R \leq \frac{R_s}{d}\}.$$

$v_{ni} =$ The non-infection terms =

$$\begin{pmatrix} \eta_1N_Q + dN_Q \\ -(1 - \alpha)\beta_1SI + \eta_2Q + \gamma Q + dQ \\ -\eta_1N_Q + \theta I + \delta_1I + dI \end{pmatrix}$$

$F_{id} =$ Jacobian of f_i at the disease-free

equilibrium point =
$$\begin{pmatrix} \beta_2S & 0 & \alpha\beta_1S \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$V_{ind} =$ Jacobian of v_{in} at the disease-free

equilibrium point =
$$\begin{pmatrix} \eta_1 + d & 0 & 0 \\ 0 & \gamma + \eta_2 + d & -(1 - \alpha)\beta_1S \\ -\eta_1 & 0 & \theta + \delta_1 + d \end{pmatrix}$$

Hence, $R_0 = \rho F_{id} V_{ind}^{-1} =$ Spectral of the matrix $= \frac{\beta_2 R_s (\theta + \delta_1 + d) + \alpha \beta_1 \eta_1 R_s}{(\mu + d)(\theta + \delta_1 + d)(\eta_1 + d)}$

5. Equilibrium Points and its Presence

The proposed model (1) consists of two positive equilibria namely,

(i) The disease-free equilibrium point $E_{de}^0 = \left\{ \frac{R_s}{(\mu + d)}, 0, 0, 0, 0, \frac{\mu R_s}{d(\mu + d)} \right\}$

(ii) The endemic equilibrium point $E_{ee}^* = (S^*, N_Q^*, Q^*, I^*, T^*, R^*)$ with

$$\begin{aligned}
 S^* &= \frac{(\mu + d)(\theta + \delta_1 + d)}{\beta_2(\theta + \delta_1 + d) + \alpha \beta_1 \eta_1}, \quad N_Q^* = \frac{(\theta + \delta_1 + d)I^*}{\eta_1}, \\
 Q^* &= \frac{(1 - \alpha)\beta_1(\theta + \delta_1 + d)I^*}{(\gamma + \eta_2 + d)[\beta_2(\theta + \delta_1 + d) + \alpha \beta_1 \eta_1]}, \\
 I^* &= \frac{\eta_1(\mu + d)(\gamma + \eta_2 + d)(R_0 - 1)}{(\gamma + \eta_2 + d)[\beta_2(\theta + \delta_1 + d) + \beta_1 \eta_1] - \beta_1(1 - \alpha)\gamma \eta_1}, \\
 T^* &= \frac{I^*[\theta(\gamma + \eta_2 + d)\{\alpha \beta_1 \eta_1 + \beta_2(\theta + \delta_1 + d)\} + \beta_1 \eta_2(1 - \alpha)(\eta_2 + d)(\theta + \delta_1 + d)]}{(\gamma + \eta_2 + d)(\epsilon + \delta_2 + d)[\beta_2(\theta + \delta_1 + d) + \alpha \beta_1 \eta_1]} \text{ and} \\
 R^* &= \frac{\mu(\gamma + \eta_2 + d)(\eta_1 + d)(\theta + \delta_1 + d)(\epsilon + \delta_2 + d)}{d(\gamma + \eta_2 + d)(\epsilon + \delta_2 + d)[\beta_2(\theta + \delta_1 + d) + \alpha \beta_1 \eta_1]} \\
 &\quad + \frac{\epsilon \theta(\gamma + \eta_2 + d)\{\alpha \beta_1 \eta_1 + \beta_2(\theta + \delta_1 + d)\}}{d(\gamma + \eta_2 + d)(\epsilon + \delta_2 + d)[\beta_2(\theta + \delta_1 + d) + \alpha \beta_1 \eta_1]} \\
 &\quad + \frac{\epsilon \beta_1 \eta_2(1 - \alpha)(\eta_1 + d)(\theta + \delta_1 + d)}{d(\gamma + \eta_2 + d)(\epsilon + \delta_2 + d)[\beta_2(\theta + \delta_1 + d) + \alpha \beta_1 \eta_1]}
 \end{aligned} \tag{1}$$

From the above expression, it is clear that if the values of the basic reproductive number are more than one ($R_0 > 1$), then the system (1) consists of a positive endemic equilibrium point. On the other hand, if the values of the basic reproductive number are less than one ($R_0 < 1$), then the system (1) does not consist of a positive endemic equilibrium point.

6 Stability Analysis of E_{de}^0 and E_{ee}^*

Theorem 6.0.1 Without the presence of disease the equilibrium point E_{de}^0 is locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$.

Proof: First I have constructed a Jacobian matrix J_0^d at E_{de}^0 of the system (1)

$$J_0^d = \begin{pmatrix}
 -(\mu + d) & -\frac{\beta_2 R_s}{(\mu + d)} & \gamma & -\frac{\beta_1 R_s}{(\mu + d)} & 0 & 0 \\
 0 & \frac{\beta_2 R_s}{(\mu + d)} - \eta_1 - d & 0 & \frac{\alpha \beta_1 R_s}{(\mu + d)} & 0 & 0 \\
 0 & 0 & -(\gamma + \eta_2 + d) & \frac{(1 - \alpha)\beta_1 R_s}{(\mu + d)} & 0 & 0 \\
 0 & \eta_1 & 0 & -(\theta + \delta_1 + d) & 0 & 0 \\
 0 & 0 & \eta_2 & \theta & -(\epsilon + \delta_2 + d) & 0 \\
 0 & 0 & 0 & 0 & \epsilon & -d
 \end{pmatrix}$$

The characteristic equation $|J_0^d - \lambda_0 I_m| = 0$ has three purely negative eigenvalues (I_m is the identity matrix) $\lambda_1^0 = -(\mu + d) < 0$, $\lambda_2^0 = -d < 0$, $\lambda_3^0 = -(\epsilon + \delta_2 + d) < 0$ and the another three eigenvalues of the equation

$$f(\lambda_0) = \lambda_0^3 + a_0 \lambda_0^2 + a_1 \lambda_0 + a_2 = 0$$

where,

$$a_0 = \frac{\alpha \beta_1 \eta_1 R_s}{(\mu + d)(\theta + \delta_1 + d)} + (\theta + \delta_1 + d) + (\gamma + \eta_2 + d)(\eta_1 + d)(1 - R_0),$$

$$a_1 = \frac{\alpha \beta_1 \eta_1 R_s (\gamma + \eta_2 + d)}{(\mu + d)(\theta + \delta_1 + d)} + [(\theta + \delta_1 + d) + (\gamma + \eta_2 + d)](1 - R_0)(\eta_1 + d) + (\theta + \delta_1 + d)(\gamma + \eta_2 + d)$$

$$a_2 = (1 - R_0)(\eta_1 + d)(\theta + \delta_1 + d)(\gamma + \eta_2 + d)$$

$$a_0 a_1 - a_2 = l m + l(1 - R_0)(\eta_1 + d)(\gamma + \eta_2 + d) + m(\theta + \delta_1 + d) > 0$$

where,

$$l = \frac{\alpha \beta_1 \eta_1 R_5}{(\mu + d)(\theta + \delta_1 + d)} + (\gamma + \eta_2 + d) + (\eta_1 + d)(1 - R_0),$$

$$m = \frac{\alpha \beta_1 \eta_1 R_5 (\gamma + \eta_2 + d)}{(\mu + d)(\theta + \delta_1 + d)} + (\theta + \delta_1 + d)(\eta_1 + d)(1 - R_0) + (\theta + \delta_1 + d)(\gamma + \eta_2 + d)$$

while, $R_0 < 1$ then both the value of $a_0 > 0, a_1 > 0$ and $a_2 > 0$.

Therefore by Routh Hurwitz criterion, the above quadratic equation has three roots with a negative real part. Hence E_{de}^0 is locally asymptotically stable (LAS) while $0 < R_0 < 1$ and unstable when $R_0 > 1$.

Theorem 6.0.2 The disease-related equilibrium point E_{ee}^* of the model system (1) is asymptotically stable locally while $R_0 > 1$.

Proof: The variation matrix J_{ee}^* at E_{ee}^* of the system (1) is given below,

$$J_{ee}^* = \begin{pmatrix} -\beta_1 I^* - \beta_2 N_Q^* - l_1 & -\beta_2 S^* & \gamma & -\beta_1 S^* & 0 & 0 \\ \alpha \beta_1 I^* + \beta_2 N_Q^* & \beta_2 S^* - m_1 & 0 & \alpha \beta_1 S^* & 0 & 0 \\ (1 - \alpha) \beta_1 I^* & 0 & -n_1 & (1 - \alpha) \beta_1 S^* & 0 & 0 \\ 0 & \eta_1 & 0 & -p_1 & 0 & 0 \\ 0 & 0 & \eta_2 & 0 & -q_1 & 0 \\ \mu & 0 & 0 & 0 & \epsilon & -d \end{pmatrix}$$

$$l_1 = (\mu + d), m_1 = (\eta_1 + d), n_1 = (\gamma + \eta_2 + d), p_1 = (\theta + \delta_1 + d) \text{ and } q_1 = (\epsilon + \delta_2 + d)$$

The characteristic equation $|J_{ee}^* - \lambda' I_d| = 0$ has two negative eigenvalues (I_d is the identity matrix)

$$\lambda'_1 = -d < 0,$$

$\lambda'_2 = -(\epsilon + \delta_2 + d) < 0$ and the other four eigenvalues of the equation

$$f(\lambda') = \lambda'^4 + e_1 \lambda'^3 + e_2 \lambda'^2 + e_3 \lambda' + e_4 = 0$$

where,

$$e_1 = (\beta_2 S^* - m_1) + (\beta_1 I^* - \beta_2 N_Q^* - l_1) - p_1 - n_1,$$

$$e_2 = \alpha \beta_1 \eta_1 S^* - p_1 n_1 + p_1 (\beta_1 I^* - \beta_2 N_Q^* - l_1) + p_1 (\beta_2 S^* - m_1) + (1 - \alpha) \beta_1 \gamma I^* + n_1 (\beta_1 I^* - \beta_2 N_Q^* - l_1) + n_1 (\beta_2 S^* - m_1) - (\beta_1 I^* - \beta_2 N_Q^* - l_1)(\beta_2 S^* - m_1),$$

$$e_3 = \alpha \beta_1 \eta_1 n_1 S^* - \alpha \beta_1 \eta_1 S^* (\beta_1 I^* - \beta_2 N_Q^* - l_1) - \beta_1 \eta_1 S^* (\alpha \beta_1 I^* + \beta_2 N_Q^*) + p_1 (1 - \alpha) \beta_1 \gamma I^* + n_1 p_1 (\beta_1 I^* - \beta_2 N_Q^* - l_1) + p_1 n_1 (\beta_2 S^* - m_1) - p_1 (\beta_1 I^* - \beta_2 N_Q^* - l_1)(\beta_2 S^* - m_1) - (1 - \alpha) \beta_1 \gamma I^* (\beta_2 S^* - m_1) - n_1 (\beta_1 I^* - \beta_2 N_Q^* - l_1)(\beta_2 S^* - m_1) + \beta_2 S^* (\alpha \beta_1 I^* + \beta_2 N_Q^*) \text{ and}$$

$$\begin{aligned}
 e_4 = & \eta_1 \gamma (1 - \alpha) \beta_1 S^* (\alpha \beta_1 I^* + \beta_2 N_Q^*) - \eta_1 (\alpha \beta_1^2 S^* I^* n_1 - \\
 & \alpha \beta_1 \beta_2 S^* N_Q^* n_1 - \alpha \beta_1 S^* n_1 l_1) - \\
 & \alpha \beta_1^2 \gamma S^* I^* \eta_1 (1 - \alpha) - \\
 & \alpha \beta_1^2 S^* I^* \eta_1 n_1 (\alpha \beta_1 I^* + \beta_2 N_Q^*) - \\
 & p_1 (1 - \alpha) \beta_1 \gamma I^* (\beta_2 S^* - m_1) - \\
 & n_1 p_1 (\beta_1 I^* - \beta_2 N_Q^* - l_1) (\beta_2 S^* - \\
 & m_1) + p_1 \beta_2 S^* (\alpha \beta_1 I^* + \beta_2 N_Q^*)
 \end{aligned}$$

As per Routh-Hurwitz Criterion if all $e_i \geq 0, i = 1, 2, 3, 4$ and $e_1 e_2 e_3 > e_3^2 + e_1^2 e_4$

then all roots ($\lambda'_3, \lambda'_4, \lambda'_5$ and λ'_6) of the above equation has negative real parts. Hence the theorem is proof.

7. Exploration of Sensitivity and Elasticity

In epidemiology, the most important thing is that the disease is either spread or not and it depends on the primary reproductive number R_0 . I am trying to find out some parameter which is more sensitive on R_0 . My desirable elasticity values of R_0 on the most sensitive parameter are given in Table: 1. With the help of the following equation I have calculated the sensitivity and elasticity values [30].

$$S_{R_0}^{s_p} = \frac{\partial R_0}{\partial s_p}$$

$$\varepsilon_{R_0}^{s_p} = \frac{\partial R_0}{\partial s_p} \times \frac{s_p}{R_0}$$

From the sensitivity and elasticity analysis, the values of R_0 will increase (decrease) if the values of the parameters $R_s, \beta_1, \beta_2,$ and α will also increase (decrease). On the other hand, the values of R_0 will increase (decrease) if the values of $\mu, \eta_1, \delta_1,$ and θ will decrease

(increase). Finally, I have found most desirable sensitive parameters are μ and θ . I neglect the second most sensitive parameter d because I assume that d is the death rate of all the population classes.

8. Analysis of the Analytical Result

In this portion, I am trying to establish my analytical result of the proposed model (1) with the assistance of some numerical techniques. Numerical Figure 1 shows that the endemic equilibrium point is locally asymptotically stable for all parameters whose values are given in Table 2. Figure 2 displays that the disease-free equilibrium point is LAS for $R_s = 0.1, \beta_2 = 0.01,$ and $\alpha = 0.018$ and other parameters values from Table 2. Figure 3 indicates the sensitive analysis results. The basic reproductive number value decreases when the values of μ and θ are increasing which is given in Figure 4. When the values of awareness (μ) increase from 0.045 to 0.165 then the values of R_0 decrease from 2.8109 to 0.8834 and the non-quarantine class, quarantine class, infected class, and treatment class is going to be extinction which is given in Figure 5, 6, 7 and 8. Figure 9 shows that all the disease-related classes will be in extinction if I increase the awareness of its value.

9. CONCLUSION

In this paper, I have discussed a six-dimensional SN_QQITR deterministic mathematical model on the COVID-19 pandemic with treatment and awareness. With the assistance of the next-generation matrix technique, I have computed the basic reproductive number R_0 . In epidemiology, the disease transmission i.e. the disease

is either spread or not depends on R_0 . From my result, it is obvious that the disease persists and spreads while $R_0 > 1$ and dies out while $R_0 < 1$. After analyzing my proposed model I have investigated two non-negative equilibrium points namely the with disease and without disease. I have studied the stability of both equilibrium points. My analytical result says that the disease-free (without disease) equilibrium point is LAS if $R_0 < 1$ and unstable while $R_0 > 1$. Also, the endemic (with disease) equilibrium point is LAS if $R_0 > 1$. I also prove the reality of my analytical findings numerically. My main motivation for this work is to decrease the disease transmission. For that reason, I have investigated some sensitive parameters with the help of sensitivity and elasticity analysis. My study indicates that the most desirable sensitive parameters are μ (awareness) θ (treatment). My numerical figure shows that if I increase the value of θ and μ then the values of R_0 must decrease. For increasing the value of μ and θ in certain values then R_0 its values go to less than 1 which ensures that the system has no disease. Finally, I conclude that the increasing of awareness and treatment can control the disease transmission.

Table 1: Sensitivity and Elasticity Analysis on R_0

Parameter	R_s	β_1	β_2	μ	α	η_1	d	δ_1	θ
s_p									
Value	0.19	0.018	0.017	0.045	0.019	0.011	0.01	0.01	0.023
$\epsilon_{R_0}^{s_p}$	1	0.005	0.99	-0.818	0.005	-0.519	-0.659	-0.001	-0.003

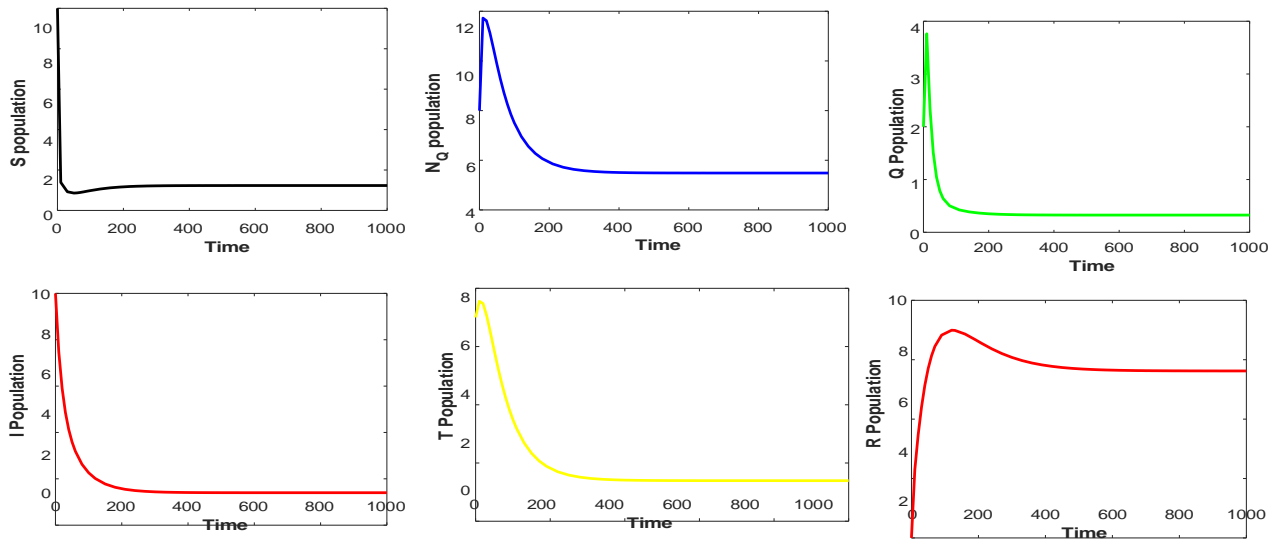


Figure 1: Stability of the endemic equilibrium point E_{ee}^*

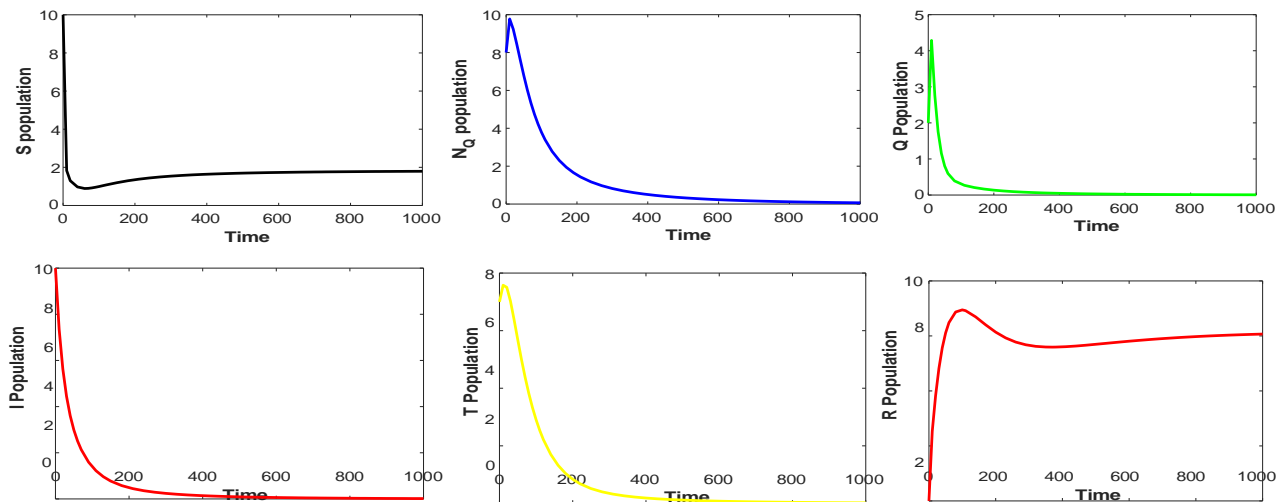


Figure 2: Stability of the disease-free equilibrium point E_{dse}^0

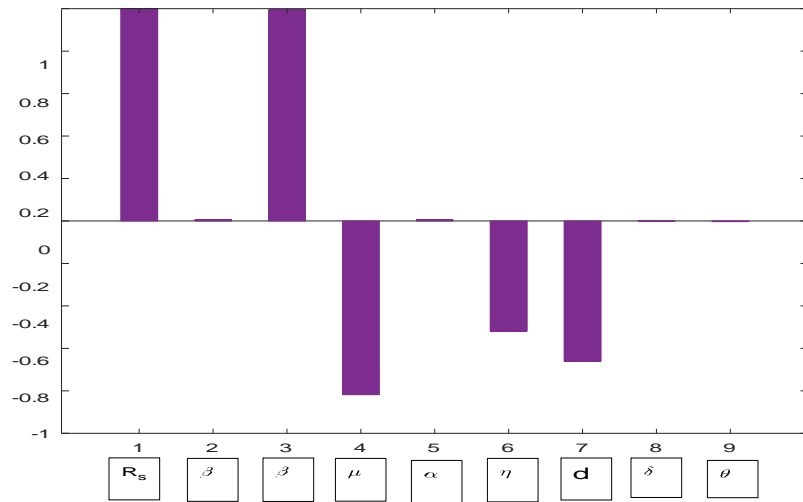


Figure 3: Elasticity on R_0 concerning the parameters

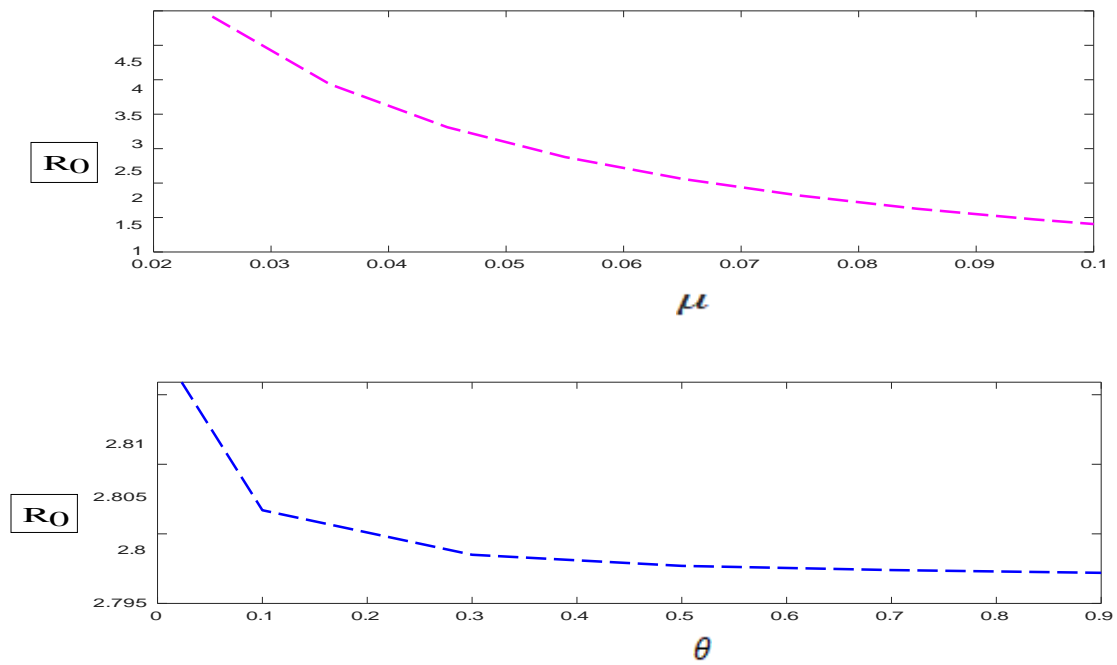


Figure 4: Relation among μ , θ and R_0

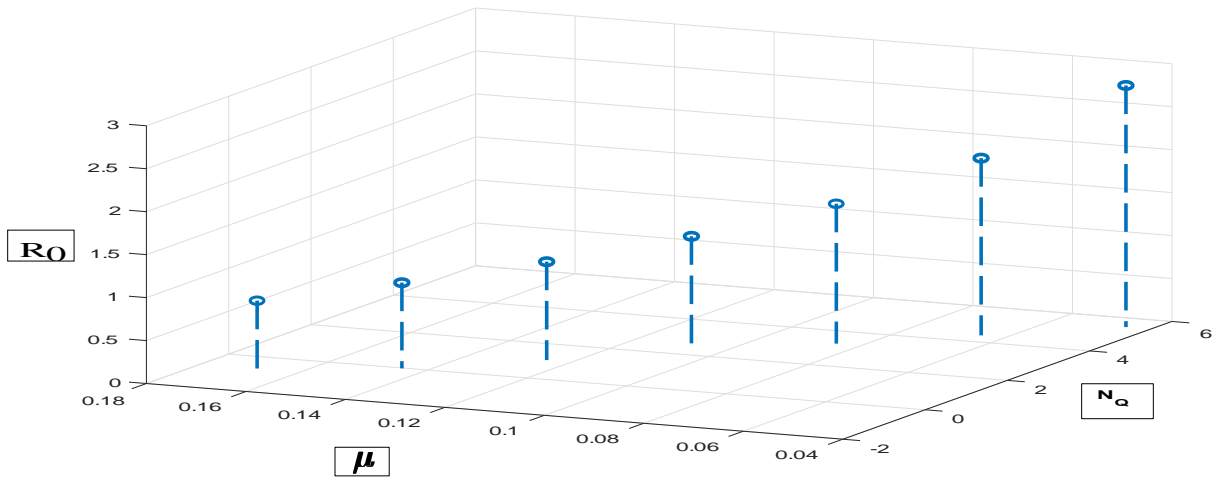


Figure 5: Relation among μ , N_0 , and R_0

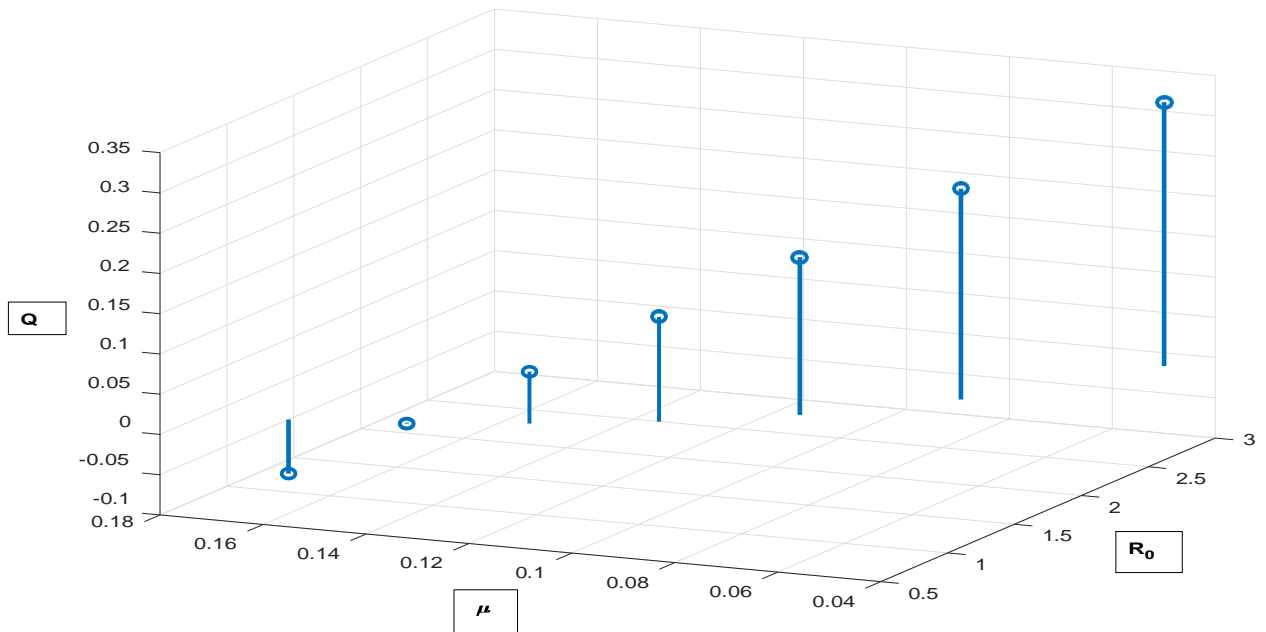


Figure 6: Relation among μ , Q , and R_0

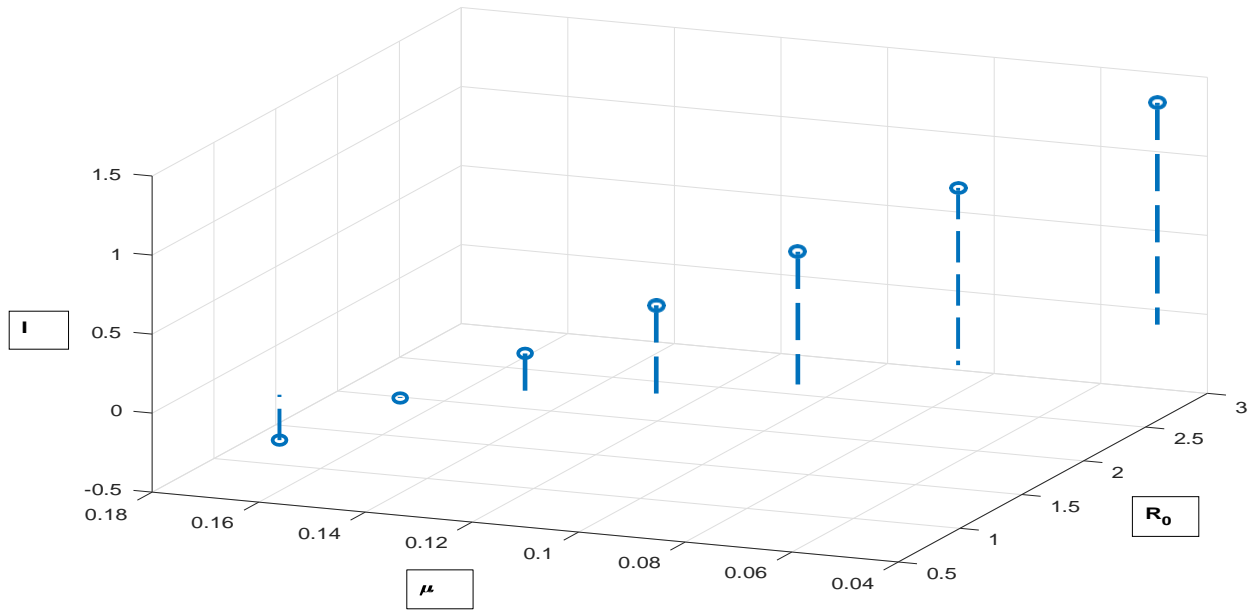


Figure 7: Relation among μ , I and R_0

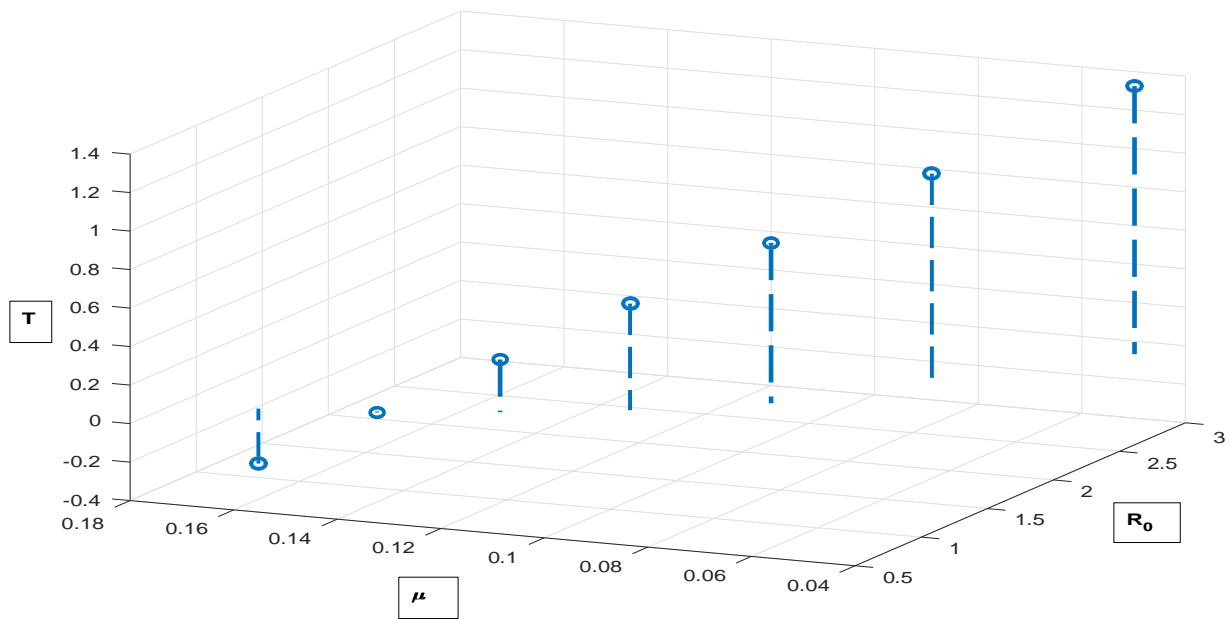


Figure 8: Relation among μ , T , and R_0

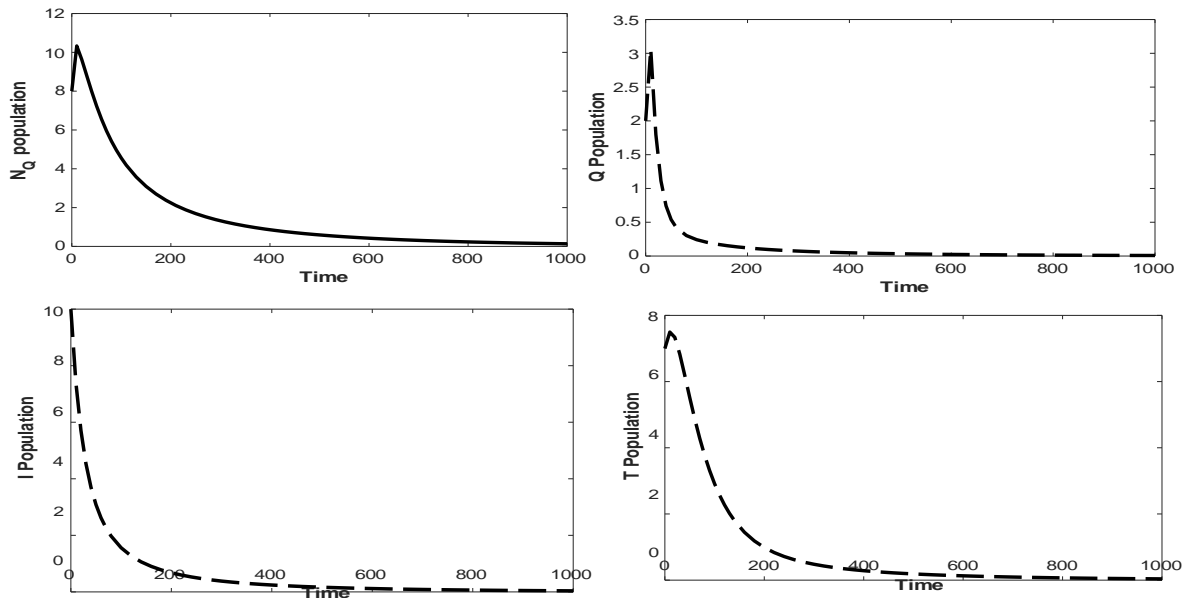


Figure 9: For increasing awareness Q , I , N_Q , and T population classes are going to extinction

Table 2: Interpretation and presumption of parameter

Symbol	Explanation	Value and source
R_s	Recruitment rate of active population	0.19 Estimated
β_1	Contact rate between S and I class	0.018 Estimated
β_2	Contact rate between S and N_Q class	0.0171 Estimated
α	A positive constant	0.019 Estimated
θ	Treatment rate of infected class	0.023
δ_1	Disease induces death rate	0.01[4]
δ_2	Disease induces death rate	0.001
d	Natural death rate of all class	0.2 Estimated
μ	Awareness rate	0.045 Estimated
η_1	Nonquarantine class goes to infected class	0.011 Estimated
η_2	Quarantine class goes to treatment class	0.012 Estimated
γ	Quarantine period	0.071 [16]
ϵ	Release rate	0.015 Estimated

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